

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Advanced Subsidiary General Certificate of Education  
Advanced General Certificate of Education**

**MATHEMATICS**

**4736**

Decision Mathematics 1

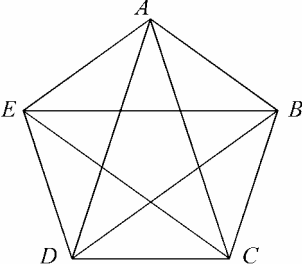
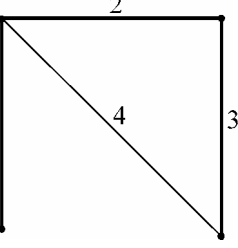
MARK SCHEME

**Specimen Paper**

<b>MAXIMUM MARK</b>	<b>72</b>
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**This mark scheme consists of 4 printed pages.**

<p><b>1</b> (i)</p>  <p><math>K_5</math> is Eulerian since every node is even</p> <hr/> <p>(ii) A path is (e.g.) <math>A-B-C</math></p> <hr/> <p>(iii) A cycle is (e.g.) <math>A-B-C-A</math></p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>For correct graph</p> <p>2 For a correct statement</p> <hr/> <p>1 For any correct path</p> <hr/> <p>1 For any correct cycle</p> <p style="text-align: center;"><b>4</b></p>
<p><b>2</b> (i) Using Kruskal's algorithm, the arc of least weight is chosen first and so is certainly included The arc of second least weight is chosen next since just two arcs cannot form a cycle</p> <hr/> <p>(ii)</p> 	<p>B1</p> <p>B1</p> <p>B1</p> <hr/> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>For identifying the first choice</p> <p>For identifying the second choice</p> <p>3 For correct justification</p> <hr/> <p>For any connected graph with 4 nodes and at least 3 arcs</p> <p>For including a cycle</p> <p>For a network having the required property</p> <p>4 For making the minimum connector clear</p> <p style="text-align: center;"><b>7</b></p>
<p><b>3</b> (i) 1st pass: <math>\underline{6} \ 3 \ 8 \ 3 \ 2</math> giving 3 6 8 3 2 2nd pass: 3 <math>\underline{6} \ 8</math> 3 2 giving 3 6 8 3 2 3rd pass: 3 6 <math>\underline{8} \ 3</math> 2 3 <math>\underline{6} \ 3</math> 8 2 <math>\underline{3} \ 3</math> 6 8 2 giving 3 3 6 8 2 4th pass: 3 3 6 <math>\underline{8} \ 2</math> 3 3 <math>\underline{6} \ 2</math> 8 3 <math>\underline{3} \ 2</math> 6 8 <math>\underline{3} \ 2</math> 3 6 8 giving 2 3 3 6 8</p> <hr/> <p>(ii) The number of operations to be carried out, and thus the time to complete the algorithm, is (approximately) proportional to the square of the number of items to be sorted</p>	<p>B1</p> <p>B1</p> <hr/> <p>M1</p> <hr/> <p>M1</p> <p>A1</p>	<p>For correct result of first pass</p> <p>For correct result of second pass</p> <hr/> <p>For correct shuttle process in third pass</p> <hr/> <p>For correct shuttle process in final pass</p> <p>5 For shuttle sort completed correctly</p> <hr/> <p>For idea of dependency on 'size' of problem</p> <p>For number of operations, or time required</p> <p>3 For square of list size</p> <p style="text-align: center;"><b>8</b></p>

4	(i)	<table border="1"> <thead> <tr> <th>STEP</th> <th>A</th> <th>B</th> <th>C</th> </tr> </thead> <tbody> <tr><td>1</td><td>6</td><td>13</td><td>0</td></tr> <tr><td>2</td><td>6</td><td>13</td><td>6</td></tr> <tr><td>4</td><td>12</td><td>6</td><td>6</td></tr> <tr><td>4</td><td>24</td><td>3</td><td>6</td></tr> <tr><td>2</td><td>24</td><td>3</td><td>30</td></tr> <tr><td>4</td><td>48</td><td>1</td><td>30</td></tr> <tr><td>2</td><td>48</td><td>1</td><td>78</td></tr> <tr><td>3</td><td>48</td><td>1</td><td>78</td></tr> <tr><td>6</td><td colspan="3">Output 78</td></tr> </tbody> </table>	STEP	A	B	C	1	6	13	0	2	6	13	6	4	12	6	6	4	24	3	6	2	24	3	30	4	48	1	30	2	48	1	78	3	48	1	78	6	Output 78			B1 M1	For assigning value to C in first Step 2 For updating A and B in first Step 4	M1 A1	For continuing algorithm and updating C For correct new value 30 for C	A1	5	For correct output
STEP	A	B	C																																														
1	6	13	0																																														
2	6	13	6																																														
4	12	6	6																																														
4	24	3	6																																														
2	24	3	30																																														
4	48	1	30																																														
2	48	1	78																																														
3	48	1	78																																														
6	Output 78																																																
		(ii)	<table border="1"> <thead> <tr> <th>STEP</th> <th>A</th> <th>B</th> <th>C</th> </tr> </thead> <tbody> <tr><td>1</td><td>A</td><td>8</td><td>0</td></tr> <tr><td>4</td><td>2A</td><td>4</td><td>0</td></tr> <tr><td>4</td><td>4A</td><td>2</td><td>0</td></tr> <tr><td>4</td><td>8A</td><td>1</td><td>0</td></tr> <tr><td>2</td><td>8A</td><td>1</td><td>8A</td></tr> <tr><td>3</td><td>8A</td><td>1</td><td>8A</td></tr> <tr><td>6</td><td colspan="3">Output 8A</td></tr> </tbody> </table>	STEP	A	B	C	1	A	8	0	4	2A	4	0	4	4A	2	0	4	8A	1	0	2	8A	1	8A	3	8A	1	8A	6	Output 8A			M1 M1 A1	For values of A doubling For values of B halving For output 8A	The output is the product of the inputs		B1	4	For identifying multiplication							
STEP	A	B	C																																														
1	A	8	0																																														
4	2A	4	0																																														
4	4A	2	0																																														
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3	8A	1	8A																																														
6	Output 8A																																																
		<b>9</b>																																															
5	(i)	A minimum connector on reduced network has arcs CE, ED, BD, AB, giving length 23 km Two shortest arcs from F have weights 7, 8 Hence lower bound is $23 + 7 + 8 = 38$ km	M1 A1 M1 A1	For attempt at a relevant minimum connector For correct weight 23 For identifying the two shortest arcs at F For showing given answer correctly	<b>4</b>																																												
		(ii)	The best upper bound is 47 km The best lower bound is 40 km	B1 B1	For the correct answer For the correct answer	<b>2</b>																																											
		(iii)	Other orders are CED, DCE, DEC, ECD, EDC Shortest is ABDCEFA, of length 42 km	M1 A1 A1	For calculation of at least one other length For any correct bound less than 47 km For the correct value 42	<b>3</b>																																											
		<b>9</b>																																															
6	(i)	<p>Least travel time is 40 minutes Route is A-B-C-D</p>	M1 M1 A1 B1	For correct use of temporary labels For updating E and D For all permanent labels correct For correct order of assignment stated	B1 B1	For correct value 40 For correct route	<b>6</b>																																										
		(ii)	The Route Inspection algorithm is used A, B, C and E are odd nodes $AB = 16$ $AC = 27$ $AE = 37$ $CE = 10$ $BE = 21$ $BC = 11$ 26           48           48 Double up on AB and CE Sum of arcs is 172 Hence shortest time is $172 + 26 = 198$ minutes	B1 B1 M1 M1 M1 A1	For stating or implying the correct algorithm For identifying the odd nodes For pairing odd nodes correctly For selecting appropriate pair for doubling For adding weights on all the arcs For correct value 198	<b>6</b>																																											
		(iii)	Nearest neighbour algorithm gives A-B-C-E-D-A Hence required path is A-B-C-E-D	M1 A1 B1	For starting the algorithm correctly, up to C For the correct cycle A-B-C-E-D-A For a correct path	<b>3</b>																																											
		<b>15</b>																																															

<p>7 (i)</p>	<p>Hence maximum <math>P = 18</math>, occurring at <math>(2, 5)</math></p>	<p>M1 M1 A1 B1✓ B1✓ B1✓ B1 B1</p>	<p>For lines <math>x + 4y = 22</math> and <math>x + y = 10</math> For line <math>-x + 2y = 8</math> For correct diagram including shading For vertices <math>(0, 0), (0, 4), (10, 0)</math> For vertex <math>(2, 5)</math> For vertex <math>(6, 4)</math> For the correct value 18 For identifying the correct vertex</p>																																																																																				
<p>(ii)</p>	<table border="1"> <thead> <tr> <th><math>P</math></th> <th><math>x</math></th> <th><math>y</math></th> <th><math>s</math></th> <th><math>t</math></th> <th><math>u</math></th> <th></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>-4</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>4</td> <td>1</td> <td>0</td> <td>0</td> <td>22</td> </tr> <tr> <td>0</td> <td>-1</td> <td>2</td> <td>0</td> <td>0</td> <td>1</td> <td>8</td> </tr> </tbody> </table> <p>Pivot on 2 in row 3</p> <table border="1"> <tbody> <tr> <td>1</td> <td>-1</td> <td>0</td> <td>0</td> <td>0</td> <td>2</td> <td>16</td> </tr> <tr> <td>0</td> <td>3</td> <td>0</td> <td>1</td> <td>0</td> <td>-2</td> <td>6</td> </tr> <tr> <td>0</td> <td><math>1\frac{1}{2}</math></td> <td>0</td> <td>0</td> <td>1</td> <td><math>-\frac{1}{2}</math></td> <td>6</td> </tr> <tr> <td>0</td> <td><math>-\frac{1}{2}</math></td> <td>1</td> <td>0</td> <td>0</td> <td><math>\frac{1}{2}</math></td> <td>4</td> </tr> </tbody> </table> <p>Now pivot on 3 in row 1</p> <table border="1"> <tbody> <tr> <td>1</td> <td>0</td> <td>0</td> <td><math>\frac{1}{3}</math></td> <td>0</td> <td><math>1\frac{1}{3}</math></td> <td>18</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td><math>\frac{1}{3}</math></td> <td>0</td> <td><math>-\frac{2}{3}</math></td> <td>2</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td><math>-\frac{1}{2}</math></td> <td>1</td> <td><math>\frac{1}{2}</math></td> <td>3</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td><math>\frac{1}{6}</math></td> <td>0</td> <td><math>\frac{1}{6}</math></td> <td>5</td> </tr> </tbody> </table> <p>Hence <math>P = 18</math> when <math>x = 2, y = 5</math></p>	$P$	$x$	$y$	$s$	$t$	$u$		1	1	-4	0	0	0	0	0	1	4	1	0	0	22	0	-1	2	0	0	1	8	1	-1	0	0	0	2	16	0	3	0	1	0	-2	6	0	$1\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	6	0	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	4	1	0	0	$\frac{1}{3}$	0	$1\frac{1}{3}$	18	0	1	0	$\frac{1}{3}$	0	$-\frac{2}{3}$	2	0	0	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	3	0	0	1	$\frac{1}{6}$	0	$\frac{1}{6}$	5	<p>B1 M1 A1 M1 M1 A1✓ M1 M1 A1 B1✓</p>	<p>For the correct pay-off row For the use of three slack variables For all constraints correct For choice of pivot For pivoting correctly For correct tableau For choice of pivot For pivoting correctly For correct tableau For reading off correctly from final tableau</p>
$P$	$x$	$y$	$s$	$t$	$u$																																																																																		
1	1	-4	0	0	0	0																																																																																	
0	1	4	1	0	0	22																																																																																	
0	-1	2	0	0	1	8																																																																																	
1	-1	0	0	0	2	16																																																																																	
0	3	0	1	0	-2	6																																																																																	
0	$1\frac{1}{2}$	0	0	1	$-\frac{1}{2}$	6																																																																																	
0	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	4																																																																																	
1	0	0	$\frac{1}{3}$	0	$1\frac{1}{3}$	18																																																																																	
0	1	0	$\frac{1}{3}$	0	$-\frac{2}{3}$	2																																																																																	
0	0	0	$-\frac{1}{2}$	1	$\frac{1}{2}$	3																																																																																	
0	0	1	$\frac{1}{6}$	0	$\frac{1}{6}$	5																																																																																	
<p>(iii)</p>	<p>Vertices <math>(0, 0) \rightarrow (0, 4) \rightarrow (2, 5)</math> indicated</p>	<p>M1 A1</p>	<p>For indication of starting at the origin For the correct correspondence indicated</p>																																																																																				

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